Intrusiveness-aware Estimation for High Quantiles of a Packet Delay Distribution

Kohei Watabe and Kenji Nakagawa
Department of Electrical Engineering, Nagaoka University of Technology
Kamitomioka 1603-1, Nagaoka, Niigata 940-2188, Japan
Email:{k_watabe, nakagawa}@vos.nagaokaut.ac.jp

***Abstract*—The active measurement of network quality, in which probe packets are injected into a network, is hindered by the intrusiveness problem, where the load of the probe traffic itself affects network quality. In this paper, we first demonstrate that there exists a fundamental bound on the accuracy of the conventional active measurement of delay. Second, to transcend that bound, we propose INTEST (INTrusiveness-aware ESTimation), an approach that compensates for delays produced by probe packets for wired networks. We show that INTEST enables an accurate high quantile estimation of delay. We do so through two simulations: a single-hop network composed of a router modeled by M/M/1 queuing, and a realistic multi-hop network modeled by a network simulator.**

I. Introduction

In network and application design, it is often necessary to accurately estimate end-to-end delay and evaluate path quality. Large end-to-end delay lowers the quality of real-time applications, such as audio/video conferencing and IP telephony. ITU-T Recommendation G.114 [1] mentions that an end-to-end delay of 150 msec or over adversely affects the communication quality of interactive Voice over IP (VoIP) applications. It is especially important to accurately estimate not only average delay, but also high quantile delays, since the communication quality of a VoIP application is characterized by them. Active measurement [2], in which probe packets are injected into a network, is one representative measurement technique for end-to-end delay. Prior work on active measurement has left us with a rich collection of literature [3], [4], [5], [6].

A problem with active measurement, however, is that when we inject many probe packets into the network (so as to improve accuracy), the load imparted by this probe traffic itself acts to impair path quality [7], [8]. In the active measurement of end-to-end delay, probe packet injection is treated as a method of sampling of the delay process of target path. Thus, were there no such intrusive effect, estimation accuracy would improve as the number of probe packets increases. As above, however, the transmission of probe packets itself consumes network resources. Thereby, the delays experienced by probe packets in a network with probe packets tend to be larger than the delays that would be experienced by normal packets in the same network without probe packets.

Needless to say, network researchers and practitioners are interested not in delay within a network with probe packets, but rather in delay within that network without probe packets. Nonetheless, because of this intrusiveness, conventional estimators based on the delay experienced by probe packets produce figures larger than the true value (i.e., the delay experienced by normal packets within the network without probe packets). In other words, conventional estimators of delay are biased. Baccelli et al. [7] indicates that the mean of a conventional estimator corresponds to network delay when injecting probe packets upon Poisson arrivals. Note, however, that under this treatment, the mean corresponds to the delay of a network with probe packets. Roughan [8] discusses accuracy as defined by variances of conventional estimators but does not focus on bias. Accordingly, discussion is needed in which such bias is taken into consideration.

In this paper, we first show that the conventional estimation of delay has a fundamental accuracy bound due to bias. To evaluate effect of bias on accuracy, we calculate Mean Squared Error (MSE) of a conventional estimator when we estimate the number of packets in a router modeled by M/M/1. Through this evaluation, we show that MSE has a lower bound across varying probing rates.

To transcend this MSE bound, we next propose INTrusiveness-aware ESTimation (INTEST), an approach that allows us to estimate the delay of a wired network without probe packets from the delay of that same network with probe packets. INTEST does this by compensating for the increased delay brought about by the load imposed by probe traffic. Performing simulations with single M/M/1 queuing and a multi-hop network, we show that INTEST enables a more accurate estimation of delay quantiles than does a conventional estimator.

The remainder of this paper is organized as follows. We formulate the intrusiveness problem of active measurement in Section II. In Section III, by theoretically calculating MSE, we show that there exists a lower bound to MSE when estimating the number of packets in a router modeled by M/M/1 queuing. In Section IV, we summarize INTEST, and in Section V, we evaluate it through simulations. Finally, we conclude our paper and present issues for future research in Section VI.

II. Formulation of the Intrusiveness Problem

First of all, we formulate the intrusiveness problem of active measurement, and show how we evaluate effect of bias on accuracy. Let us consider a network with some network edges. We define the target traffic as traffic that streams from an edge a to an edge b. We let $D_{\mathrm{g}}(t)$ denote a delay that is experienced by a packet that is enqueued to a queue of source side edge a at time t. The subscript g indicates that the process represents the so-called ground truth of the delay (i.e., the delay is not affected by probe load). We define cross traffic as the traffic that excludes the target traffic from the whole traffic in the network. If applying active measurement to determine characteristic of $D_{\mathrm{g}}(t)$, we add probe traffic to the traffic that streams from the edge a to the edge b.

On the network with probe packets, a packet of traffic that streams from the edge a to the edge b experiences a delay $D_{\mathrm{gp}}(t)$ [sec], where the subscript p stands for probe traffic. Note that $D_{\mathrm{gp}}(t)$ differs from $D_{\mathrm{g}}(t)$, even if cross traffic is unchanged.

Based on delays that are experienced by n probe packets, we estimate mean, quantile, and other parameters descriptive of end-to-end delay. We let $T_i\,(i = 1,\ldots,n)$ denote time at which ith probe packet is injected into the network (i.e., the packet is enqueued to a queue of source side edge). The delay that is experienced by ith probe packet is expressed by $D_{\mathrm{gp}}(T_i)$. When estimating mean delay, the conventional estimator is

$$\hat{D}_{\mathrm{ave}} = \frac{1}{n}\sum_{i=1}^{n} D_{\mathrm{gp}}(T_i). \tag{1}$$

Furthermore, when we estimate q-quantile of delay, we define $\kappa(q) = \lceil (1-q)n \rceil$, where $\lceil\cdot\rceil$ denotes ceiling function. Here, the conventional estimator is taken as the $\kappa(q)$th largest delay among the delays $D_{\mathrm{gp}}(T_i)$ experienced by the probe packets.

Since $D_{\mathrm{gp}}(t) \neq D_{\mathrm{g}}(t)$, the above estimators are generally not unbiased. A delay experienced by a probe packet is not much affected by other probe packets when the number of probe packets n is small. However, when n is small, estimator variance is large (and accurate estimation is difficult) since there are only a few samples. On the other hand, the accuracy of the estimator increasingly suffers from bias if we increase the number of samples because of the delays experienced by other packets. In short, there is a trade-off between the variance and bias of an estimator.

A biased estimator can be assessed by MSE, which is a statistic composed of the variance and bias of an estimator. The MSE of an arbitrary estimator $\hat{P}$ is defined as follows:

$$\mathrm{Var}[\hat{P}] + \{\mathrm{E}[\hat{P}] - P^*\}^2 = \mathrm{E}[(\hat{P} - P^*)^2], \tag{2}$$

where P^* denotes the true value of the target delay.

III. A Fundamental Bound on the Accuracy of Active Measurement

By analyzing a router modeled by M/M/1 queuing, we show that the conventional estimation by active measurement of delay is restricted by a fundamental bound on accuracy due to bias. Here, we evaluate the accuracy of estimation by MSE and then, by plotting the MSE of the estimator as a function of probing rate, verify the degree to which accuracy improves with higher probing rates.

To theoretically verify the dependence of accuracy on probing rate while taking such bias into consideration, we consider MSE upon an estimation of the number of packets (not delay) in a router under an M/M/1 queuing model. We assume that probe packets and target traffic packets are generated according to a Poisson arrival, and follow an exponential distribution with mean service time $1/\mu$. Here, n probe packets are injected with rate λ_{p}, and the sending rate of target traffic is λ_{g}. We estimate the mean number of packets in the router without probe packets by an arithmetical mean $\hat{M}_{\mathrm{ave}} = n^{-1}\sum_{i=1}^{n} M_{\mathrm{gp}}(T_i)$, where $M_{\mathrm{gp}}(T_i)$ denotes the number of packets in the router at time T_i. Under active measurement, we cannot practically determine the number of packets in a router by injecting probe packets. A verification of the number of packets in a router can, however, provide good insights on accuracy characteristics since we can analytically derive the MSE.

We can express the autocovariance function of the process of the number of packets in a router by primary functions as follows [8]:

$$r(\rho,\tau) \simeq \frac{\rho}{2(1-\rho)^2}\left(\mathrm{e}^{-A(\rho)|\tau|} + \mathrm{e}^{-B(\rho)|\tau|}\right),$$
$$A(\rho) = \frac{(1-\rho)^2}{1+\rho+\sqrt{\rho}}, \qquad B(\rho) = \frac{(1-\rho)^2}{1+\rho-\sqrt{\rho}},$$

where ρ denotes utilization.

The variance $\sigma^2 = \mathrm{Var}[\hat{M}_{\mathrm{ave}}]$ that corresponds to $\mathrm{Var}[\hat{P}]$ in Eq. (2) can be expressed by $r(\rho,\tau)$ and $\rho_{\mathrm{gp}} \equiv (\lambda_{\mathrm{g}} + \lambda_{\mathrm{p}})/\mu$ as

$$\begin{aligned}\sigma^2 &= \frac{1}{n}\mathrm{Var}[r(\rho_{\mathrm{gp}},0)] + \frac{1}{n^2}\sum_{i\neq j} r(\rho_{\mathrm{gp}}, |T_i - T_j|)\\ &= \frac{1}{n}\mathrm{Var}[r(\rho_{\mathrm{gp}},0)] + \frac{1}{n^2}\sum_{i\neq j}\int_0^{\infty} r(\rho_{\mathrm{gp}},t) f(t,|i-j|,1/\lambda_{\mathrm{p}})\mathrm{d}t\\ &\simeq \frac{\rho_{\mathrm{gp}}}{n(1-\rho_{\mathrm{gp}})^2} + \frac{2}{n^2}\sum_{k=1}^{n}\left\{\frac{\rho_{\mathrm{gp}}(n-k)}{2(1-\rho_{\mathrm{gp}})}\right.\\ &\qquad \left.\times\left(\frac{(\lambda_{\mathrm{p}})^k}{\lambda_{\mathrm{p}} + A(\rho_{\mathrm{gp}})^k} + \frac{(\lambda_{\mathrm{p}})^k}{\lambda_{\mathrm{p}} + B(\rho_{\mathrm{gp}})^k}\right)\right\},\end{aligned}$$

where $f(t,k,\alpha)$ denotes the probability density function of an Erlang distribution with shape parameter k and scale parameter α. On the other hand, the bias ε that corresponds to $\mathrm{E}[\hat{P}] - P^*$ in Eq. (2) can be expressed in terms of $\rho_{\mathrm{g}} \equiv \lambda_{\mathrm{g}}/\mu$ and $\rho_p \equiv \lambda_{\mathrm{p}}/\mu$ as [8]

$$\varepsilon = \frac{\rho_{\mathrm{gp}}}{1-\rho_{\mathrm{gp}}} - \frac{\rho_{\mathrm{g}}}{1-\rho_{\mathrm{g}}} = \frac{\rho_p}{(1-\rho_{\mathrm{g}})(1-\rho_{\mathrm{gp}})}.$$

As a result, we can calculate MSE as $\sigma^2 + \varepsilon^2$.

Substituting concrete values into the MSE variables and varying the number n of probe packets, we derive the MSE relation shown in Fig. 1. Assuming link capacity and mean packet length to be 155.52 Mbps and 600 byte, respectively, we set μ of M/M/1 to 32400 packet/sec. We set the mean measurement period l to 1.0 sec. To inject n probe packets in the period, we set probing rate λ_{p} to n/l. We will finish the measurement by the nth probe packet. Note that injecting time of the nth probe packet is not always l since injecting

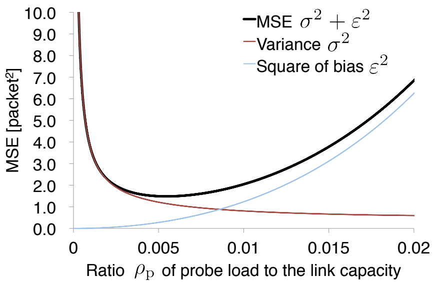

Fig. 1. MSE of an estimator for the number of packets in a router modeled by M/M/1 queuing

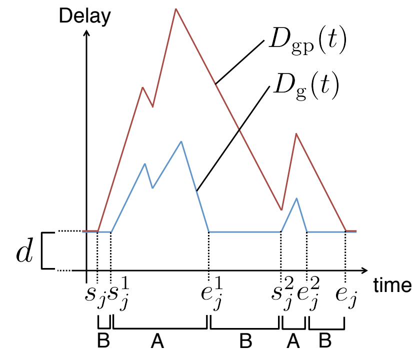

Fig. 2. Relationship between $D_{\rm g}(t)$ and $D_{\rm gp}(t)$

time follows Poisson arrivals though the mean time is l. Note that in Fig 1, the horizontal axis represents the ratio of probe load to the link capacity.

In Fig. 1, we see that MSE has a lower bound (i.e., a limit to accuracy). When the ratio $\rho_{\rm p}$ of probe load to the link capacity is low, the variance σ^2 is large, hence MSE is large. This is because we cannot obtain sufficient sampling. When the ratio of probe load to the link capacity is large, MSE also becomes large, even though the variance is low, because the bias ε is large. The lower bound of MSE is around 1.5 packet^2. We cannot obtain a more accurate estimation by increasing nor decreasing the number of probe packets. Furthermore, in practical measurements, it is very difficult to determine the optimal number of probe packets because of a dependency on the ratio $\rho_{\rm g}$ of traffic load to link capacity. The process of the number of packets in a router may not correspond to that of delay, as mentioned above. The delay is, however, proportional to the number of packets, and we confirm that similar results are obtained upon delay estimation based simulation (see Section V).

IV. Intrusiveness-aware Estimation

We here propose INTEST, an approach that compensates for delays produced by probe packets on wired networks and, by that, transcends the accuracy bound mentioned in Section III. INTEST estimates the delay $D_{\rm g}(t)$ of the network without probe packets from the delay $D_{\rm gp}(T_i)$ of the network with probe packets. It does this by subtracting the delay produced by the load imparted by probe traffic. Note that $D_{\rm gp}(t)$ is always larger than $D_{\rm g}(t)$ as shown in Fig. 2.

We first consider a network composed of single router with FIFO queuing, and we clarify the relationship between delay $D_{\rm g}(t)$ of the network without probe packets and delay $D_{\rm gp}(t)$ of the network with probe packets. Letting c [bps] denote link capacity, we define the amount of data $B_{\rm gp}(t)$ [bit] in queue at time t as

$$B_{\rm gp}(t) = \begin{cases} \displaystyle\lim_{\tau \to t-0} \sum_{h=1}^{M_{\rm gp}(\tau)} x_h - c(t - u_0(t)), & (M_{\rm gp}(t) > 0), \\ 0\,, & (M_{\rm gp}(t) = 0), \end{cases}$$

where x_h [bit] ($h = 1, 2, \ldots, M_{\rm gp}(t)$) denotes the length of hth packets in the queue and $u_0(t)$ denotes the transmission start time of a packet transmitting at time t. A delay $D_{\rm gp}(t)$ that is experienced by a packet sent at time t is related to $B_{\rm gp}(t)$ as $D_{\rm gp}(t) = B_{\rm gp}(t)/c + d$, where d denotes a propagation delay.

Letting an interval $[s, e)$ denote a busy period (i.e., a maximal interval where the number of packets in the router is positive), amounts of data $B_{\rm gp}(t_1)$ and $B_{\rm gp}(t_2)$ at times t_1 and t_2 ($s \le t_1 \le t_2 < e$) are related as follows.

$$B_{\rm gp}(t_2) = B_{\rm gp}(t_1) + X_{\rm gp}(t_1, t_2) - c(t_2 - t_1),$$

where $X_{\rm gp}(t_1, t_2)$ denotes the total amount of traffic arrived in interval $[t_1, t_2)$. We let $[s_j, e_j)$ denote the start and end time of the jth busy period of a network with probe packets (see Fig. 2). We derive the following equation regarding the amount of data $B_{\rm gp}(t)$ [bit] in the network with probe packets.

$$B_{\rm gp}(T_i) = B_{\rm gp}(T_{i-1}) + X_{\rm g}(T_{i-1}, T_i) + x^{\rm p}_{i-1} - c(T_i - T_{i-1}),$$
$$(\, i \in \{\, x \,|s_j \le T_{x-1}, T_x < e_j \,\}).$$

Here, $X_{\rm g}(t_1, t_2)$ [bit] denotes the total amount of cross and target traffic in an interval $[t_1, t_2)$ and $x^{\rm p}_i$ [bit] denotes the length of ith probe packet. Note that $B_{\rm gp}(T_i)$ does not include the data of a probe packet sent at time T_i.

We can clarify relationship between $D_{\rm g}(T_i)$ and $D_{\rm gp}(T_i)$ since a similar relationship holds for $B_{\rm g}(t)$ [bit] in $[s^k_j, e^k_j)\,(k = 1, 2, \ldots, m_j)$ (intervals A in Fig. 2). Here, we let $s^k_j (k = 1, 2, \ldots, m_j)$ and e^k_j denote the start and end times of the kth busy period of the network without probe packets within an interval $[s_j, e_j)$. Then, we have

$$B_{\rm g}(T_i) = B_{\rm g}(T_{i-1}) + X_{\rm g}(T_{i-1}, T_i) - c(T_i - T_{i-1}),$$
$$B_{\rm g}(T_i) = B_{\rm g}(T_{i-1}) + B_{\rm gp}(T_i) - B_{\rm gp}(T_{i-1}) - x^{\rm p}_{i-1},$$
$$D_{\rm g}(T_i) = D_{\rm g}(T_{i-1}) + D_{\rm gp}(T_i) - D_{\rm gp}(T_{i-1}) - \frac{x^{\rm p}_{i-1}}{c} \quad (3)$$

for $i \in \{\, x \,|s^k_j \le T_{x-1}, T_x < e^k_j \,\}$. Note that each busy period for the network without probe traffic is included in a busy period for the network with probe traffic.

Moreover, since the right hand side of Eq. (3) can be expressed by $d - (T_i - T_{i-1})$ when T_i and T_{i-1} are in $[s_j, s^1_j)$,

$[e_j^k, s_j^{k+1})$ $(k = 1, 2, \ldots, m_j - 1)$ or $[e_j^{m_j}, e_j)$ (intervals B in Fig. 2), the equation,

$$D_{\mathrm{g}}(T_i) = \max\left(D_{\mathrm{g}}(T_{i-1}) + D_{\mathrm{gp}}(T_i) - D_{\mathrm{gp}}(T_{i-1}) - \frac{x_{i-1}^{\mathrm{p}}}{c}, d\right), \quad (4)$$

holds for all i that do not satisfy $s_j < T_{i-1} \leq s_j^k < T_i < e_j$ or $s_j < T_{i-1} \leq e_j^k < T_i < e_j$. Eq. (4) means that we can estimate $D_{\mathrm{g}}(T_i)$ from delays $D_{\mathrm{gp}}(T_i)$ and $D_{\mathrm{gp}}(T_{i-1})$ experienced by probe packets.

We consider that Eq. (4) approximately holds when i satisfies $s_j < T_{i-1} \leq s_j^k < T_i < e_j$ or $s_j < T_{i-1} \leq e_j^k < T_i < e_j$, although we cannot express $D_{\mathrm{g}}(T_i)$ unless we use traffic in an interval $[T_{i-1}, T_i)$ as a function of time t. The approximation is reasonable when we can assume that the traffic including cross and target traffic have a constant rate $X(T_{i-1}, T_i)/(T_i - T_{i-1})$ in an interval $[T_{i-1}, T_i)$.

In practical measurements, we must estimate a start time s_j of a busy period, an end time e_j of a busy period, a propagation delay d, and link capacity c. They are also possible to obtain $D_{\mathrm{gp}}(T_i)$ as a delay experienced by a probe packet. By using threshold δ, we can detect start of busy periods by $U = \{T_i \,|\, D_{\mathrm{gp}}(T_{i-1}) \leq d + \delta < D_{\mathrm{gp}}(T_i)\}$. Hence, $\hat{s}_j$ that is a jth smallest element of U is an estimator of s_j. Similarly, $\hat{e}_j$ that is jth smallest element of $V = \{T_i \,|\, D_{\mathrm{gp}}(T_{i-1}) \geq d + \delta > D_{\mathrm{gp}}(T_i)\}$ is an estimator of e_j. When the probing rate is λ_{p}, the threshold δ need not be longer than $1/\lambda_{\mathrm{p}}$, because the amount of data in a router has been never 0 in the interval $[t, t + 1/\lambda_{\mathrm{p}}]$ if delay at time t is greater than $1/\lambda_{\mathrm{p}}$. A propagation delay d can be estimated by $\hat{d}$, the minimum delay of delays experienced by probe packets. With regards to c, the following equation holds:

$$\frac{x_i^{\mathrm{p}} + X_{\mathrm{g}}(T_{i-1}, T_i)}{c} = (T_i + D_{\mathrm{gp}}(T_i)) - (T_{i-1} + D_{\mathrm{gp}}(T_{i-1})).$$

Since $X_{\mathrm{g}}(T_{i-1}, T_i) \geq 0$, the estimator $\hat{c}$ of c is

$$\frac{x_i^{\mathrm{p}}}{\hat{c}} = \min_{2 \leq i \leq n} ((T_i + D_{\mathrm{gp}}(T_i)) - (T_{i-1} + D_{\mathrm{gp}}(T_{i-1})))$$

$$\hat{c} = \max_{2 \leq i \leq n} \left(\frac{x_i^{\mathrm{p}}}{T_i - T_{i-1} + D_{\mathrm{gp}}(T_i) - D_{\mathrm{gp}}(T_{i-1})}\right). \quad (5)$$

Based on Eq. (4), $\hat{s}_j$, $\hat{e}_j$, $\hat{d}$, and $\hat{c}$, we can estimate a delay $D_{\mathrm{g}}(T_i)$ of a network without probe packets from a delay $D_{\mathrm{gp}}(T_i)$ of a network with probe packets. Let $\hat{D}_{\mathrm{g}}(T_i)$ denote an estimator for $D_{\mathrm{g}}(T_i)$. Considering $\hat{D}_{\mathrm{g}}(T_i) = D_{\mathrm{gp}}(T_i)$ if T_i is in an idle period (i.e., a maximal interval where the number of packets in a router is 0), the estimator of $D_{\mathrm{g}}(T_i)$ $(1 < i \leq n)$ is given by

$$\hat{D}_{\mathrm{g}}(T_i) = \begin{cases} \max\left(\hat{D}_{\mathrm{g}}(T_{i-1}) + D_{\mathrm{gp}}(T_i) - D_{\mathrm{gp}}(T_{i-1}) - \frac{x_{i-1}^{\mathrm{p}}}{\hat{c}}, \hat{d}\right), & \hat{s}_j \leq T_i \leq \hat{e}_j \\ D_{\mathrm{gp}}(T_i), & \text{otherwise.} \end{cases} \quad (6)$$

According to the definition of $\hat{s}_j$, $D_{\mathrm{g}}(T_i) = D_{\mathrm{gp}}(T_i)$ when $i = 1$. We can estimate an average delay of a network without probe packets by replacing $D_{\mathrm{gp}}(T_i)$ with $\hat{D}_{\mathrm{g}}(T_i)$ in Eq. (1). The $\kappa(q)$th largest value within the values of $\hat{D}_{\mathrm{g}}(T_i)$ is the q-quantile estimator of a network without probe packets.

INTEST can be used to estimate delay in multi-hop networks composed of multiple routers, using timestamps provided by each router. Letting T_i^k denote a timestamp of kth router recorded on ith probe packet, we can estimate a queuing delay of kth router by $D_{\mathrm{gp}}^k(T_i) = T_i^k - T_{i-1}^k - \min_{2 \leq i \leq n}(T_i^k - T_{i-1}^k)$. Replacing $D_{\mathrm{gp}}(T_i)$ in Eqs. (5) and (6) with $D_{\mathrm{gp}}^k(T_i)$ and setting $\hat{d}$ in Eq. (6) to 0, we can estimate a queuing delay of kth router without probe packets. By calculating the sum of the queuing delays of each router and $\hat{d}$, we can estimate the end-to-end delay of a multi-hop network composed of multiple routers. Note that time synchronization across routers is not required to make this estimate.

V. Evaluation

We next evaluate the performance of INTEST through simulations to confirm that it can produce more accurate estimates than conventional active measurement. Through simulation, we first evaluate INTEST in a single-hop network composed of a single router. We then move on to a valuation of INTEST as applied to a multi-hop network. Because the theoretical value of MSE of an estimator can be derived in M/M/1 queueing, we estimate the number of packets in a router in addition to delay when evaluating of single-hop network.

A. A Network Composed of Single Router

We evaluate INTEST in a single-hop network composed of a router that is modeled by M/M/1 queuing, and show that INTEST can produce more accurate estimations than can conventional active measurement. So as to clarify the fundamental characteristics of INTEST through a simple scenario, we focus here on average number of packets in a router and on quantiles of delay.

We first perform a simulation under an assumption that we can determine the number of packets in a router at the time of the probe packet injection. The simulation is useful in evaluating the fundamental characteristics of INTEST. Note that in practice, we cannot determine the number of packets in a router by a probe packet injection by measurement. We can, however, theoretically calculate the MSE, as described above. Parameters for this simulation are the same as those for the simulation shown in Section III.

To estimate the number of packets in a router, we modify the INTEST estimator of Eq. (6) as follows.

$$\hat{M}_{\mathrm{g}}(T_i) = \begin{cases} \max\left(\hat{M}_{\mathrm{g}}(T_{i-1}) + M_{\mathrm{gp}}(T_i) - M_{\mathrm{gp}}(T_{i-1}) - 1, 0\right), & \hat{s}'_j \leq T_i \leq \hat{e}'_j \\ M_{\mathrm{gp}}(T_i), & \text{otherwise,} \end{cases}$$

where $\hat{M}_{\mathrm{g}}(t)$ [packet] is the estimator of the number of packets in a router without probe packets at time t. Also, $\hat{s}'_j$ and $\hat{e}'_j$ that are jth smallest elements of $U' = \{T_i \,|\, M_{\mathrm{gp}}(T_{i-1}) <$

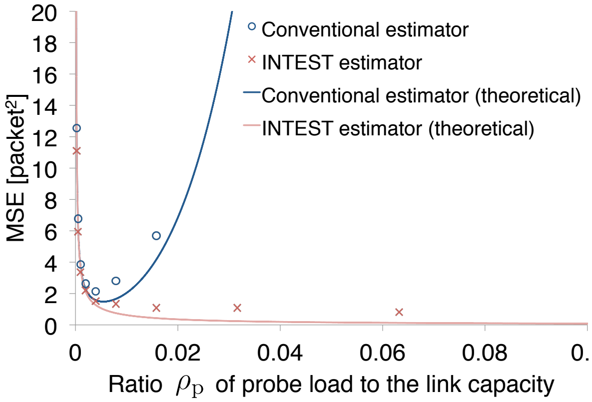

Fig. 3. MSEs of the conventional and INTEST estimators when we estimate the number of packets in a router modeled by M/M/1 queuing

$\delta \leq M_{\mathrm{gp}}(T_i)\}$ and $V' = \{T_i \,|\, M_{\mathrm{gp}}(T_{i-1}) > \delta \geq M_{\mathrm{gp}}(T_i)\}$ are estimators of s'_j and e'_j, respectively. We set threshold δ to 7.5 packets.

Upon changing the number of probe packets for each simulation from 2 packets to 2048 packets, we calculate MSE for the conventional estimator and INTEST estimator of Eq. (6). Results are shown in Fig. 3. In the calculation of MSE, we repeated the simulation 5,000 times at each number of probe packets. The theoretical values for the conventional estimator in Fig. 3 are the same as the MSE values shown in Fig. 1, and the theoretical values for the INTEST estimator are σ^2, which is the MSE for an unbiased estimator. We can confirm from Fig. 3 that the minimum value of the MSE of the INTEST estimator is smaller than that of the conventional estimator. This shows that INTEST achieves highly accurate measurement beyond the bound of conventional active measurement.

Next, we evaluate network delay under the same conditions as the above simulation. To confirm that $\hat{D}_{\mathrm{g}}(T_i)$—a sample of a delay process obtained by compensation with INTEST in Eq. (6)—corresponds to the delay of the network without probe packets at time T_i, we derive a delay process of the network without probe packets and compare $\hat{D}_{\mathrm{g}}(T_i)$ to it. We set the threshold δ to 0.2 msec when deriving $\hat{D}_{\mathrm{g}}(T_i)$. In Fig. 4, we show $\hat{D}_{\mathrm{g}}(T_i)$ and delay processes over the interval [0.02, 0.04) sec. In the example, the number n of probe is 1024. From the figure, we note that samples $\hat{D}_{\mathrm{g}}(T_i)$ of the INTEST estimator are very close to delay process $D_{\mathrm{g}}(t)$ of the network without probe packets, although samples $D_{\mathrm{gp}}(T_i)$ of the conventional estimator differ greatly from the process.

Repeating similar simulations 5,000 times for each number of probe packets, we derive the bias and MSE of the estimator at 95%-quantile of delay and evaluate the accuracy. It is well known that the cumulative distribution function of M/M/1 queuing delay is

$$F(t) = 1 - \frac{\lambda}{\mu}\mathrm{e}^{-(\mu-\lambda)t}, \qquad (0 \leq t).$$

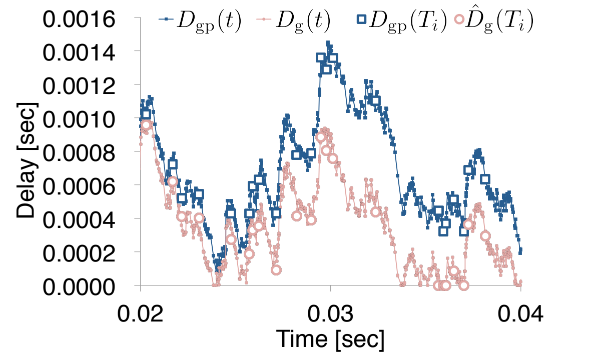

Fig. 4. A comparison of a delay process of the network without probe packets and samples $\hat{D}_{\mathrm{g}}(T_i)$ by INTEST

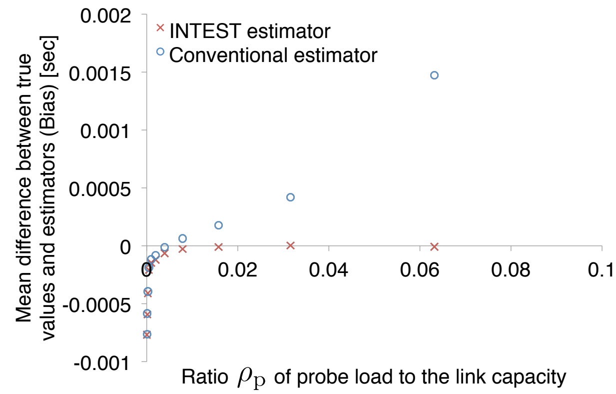

Fig. 5. Biases of the conventional and INTEST estimators when we estimate 95%-quantile of delay of a router modeled by M/M/1 queuing

Then, the true value $D^*_{\mathrm{qua}}(q)$ of q-quantile is

$$D^*_{\mathrm{qua}}(q) = \begin{cases} -\dfrac{1}{\mu-\lambda}\log\dfrac{\mu}{\lambda}(1-q), & 1-\dfrac{\lambda}{\mu} \leq q \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

We show the bias (i.e., mean difference between the value of Eq. (7) and an estimator) of the conventional and INTEST estimator in Fig. 5. We find that the INTEST estimator can provide an unbiased estimation when the number of probe packets sufficiently large. The conventional estimator and INTEST estimator both show a bias when the ratio of probe load to the link capacity is small. This is because the number of probe packets, at 2 to 64 packets, is too small. As for MSE, we obtained results similar to those shown in Fig. 3.

From the results of the above simulations, we confirm that INTEST goes beyond the fundamental accuracy bound pointed out in Section III and enables highly accurate measurements of delay in a single-hop network. In INTEST, when the probing rate increases, accuracy does not suffer but instead improves. In conventional estimation, the parameter area (in which both bias and variance are small) is itself very small. With INTEST, however, we can conduct unbiased and small variance estimations as long as the probing rate is fairly high.

B. A Network Composed of Multiple Routers

To confirm that INTEST enables accurate estimation for a network composed of multiple routers, we perform a simulation with an ns-3 simulator [9]. INTEST can estimate end-to-end delay of a network that is composed of multiple routers by recording the timestamps of routers along the target path. We perform a simulation of the network shown in Fig. 6 with an ns-3 simulator. From the results, we then estimate the end-to-end delay of the network without probe packets from the end-to-end delay experienced by probe packets. The capacity of bottleneck links (i.e., N_1-N_2 and N_2-N_3) is taken as 15.552 Mbps, and that of the other links as 62.208 Mbps. In the simulation, the target traffic is streamed from node N_0 to node N_4. The packet length of the target traffic is 600 bytes. The sending time of the packets follow a Poisson arrival; here, we tuned the sending rate so as to occupy 10% of the link capacity of the bottleneck links. As for cross traffic, traffic streams along two routes (i.e., from N_5 to N_6 and from N_6 to N_7), and there are two flows for each route. The cross traffic flows over repetitive ON/OFF intervals (mean 0.5 sec), with the constant bit rate of 8 Mbps during ON periods. When two flows sharing the same route are both in an ON interval, the total amount of traffic exceeds the bottleneck link capacity, thereby producing packet delays on node N_1 or N_2. Packet loss does not occur since the size of the buffers on node N_1 or N_2 is supposed to be sufficiently large. Probe traffic is streamed from P_s to P_d, and the size of the probe packets is 64 byte. The injecting time of the probe packets follows a Poisson arrival, and we tuned the probing rate so as to occupy 0.125%, 0.25%, 1.0%, 2.0%, and 4.0% of the link capacity of the bottleneck links. All of the packets in the simulation are UDP packets, and the simulation time is 10.0 sec. On an assumption that the link capacity is already known, we did not estimate the link capacity with Eq. (5).

In Fig. 7, we show the differences between estimators and the true value of 95%-quantile end-to-end delay. From the figure, we note that the INTEST estimators are very closed to the true value while the conventional estimators become increasingly separated from the true value with an increasing ratio of probe load to link capacity. Through this simulation, we confirm that INTEST can accurately estimate the end-to-end delay of a network composed of multiple routers.

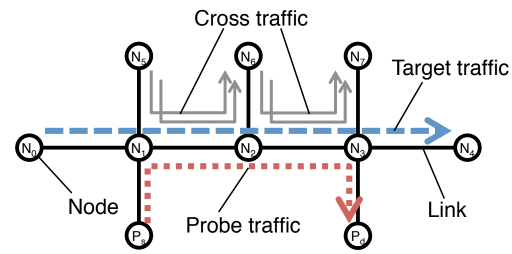

Fig. 6. Simulation model

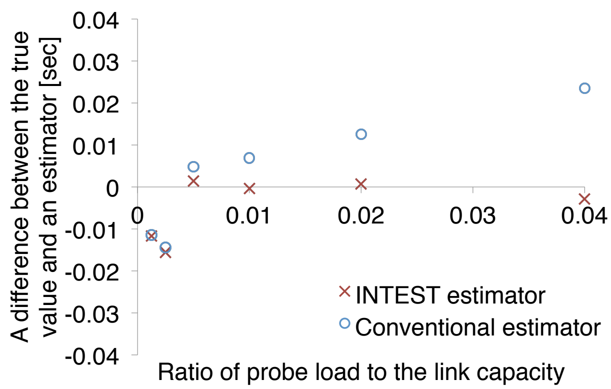

Fig. 7. Estimation by INTEST and conventional estimators of end-to-end delay of a network composed of multiple routers

VI. Conclusion

In this paper, we demonstrated that there exists a fundamental accuracy bound to conventional active measurement of delay and proposed INTEST as a means to take us beyond that bound. We evaluated accuracy in terms of MSE, taking bias into consideration in the case of an estimation of the number of packets in a router modeled by M/M/1 queuing, and showed that MSE has a lower bound across varying probing rates. Performing simulations of a single M/M/1 queuing and a multi-hop network, we demonstrated that our INTEST estimator provides unbiased and small variance estimation of high quantile end-to-end delay, whereas the conventional estimator does not provide unbiased estimation.

We plan to evaluate INTEST on a real network and extend it to packet loss estimation and to wireless networks in future works.

Acknowledgment

This work was partly supported by JSPS Grant-in-Aid for Young Scientists (Start-up) Number 26880008.

References

[1] "One-way Transmission Time," *ITU-T Recommendation G.114*, May 2003.

[2] S. Lee, K. Levanti, and H. S. Kim, "Network Monitoring: Present and Future," *Computer Networks*, vol. 65, no. 2, pp. 84–98, Jun. 2014.

[3] J.-C. Bolot, "Characterizing End-to-End Packet Delay and Loss in the Internet," *Journal of High Speed Networks*, vol. 2, no. 3, pp. 289–298, 1993.

[4] L. De Vito, S. Rapuano, and L. Tomaciello, "One-way Delay Measurement: State of the Art," *IEEE Transactions on Instrumentation and Measurement*, vol. 57, no. 12, pp. 2742–2750, Dec. 2008.

[5] H. X. Nguyen and M. Roughan, "Rigorous Statistical Analysis of Internet Loss Measurements," *IEEE/ACM Transactions on Networking*, vol. 21, no. 3, pp. 734–745, Jun. 2013.

[6] J. Strauss, D. Katabi, and F. Kaashoek, "A Measurement Study of Available Bandwidth Estimation Tools," in *Proceedings of the 3rd ACM SIGCOMM Conference on Internet Measurement (IMC 2003)*, Miami, FL, USA, Oct. 2003, pp. 39–44.

[7] F. Baccelli, S. Machiraju, D. Veitch, and J. Bolot, "The Role of PASTA in Network Measurement," *ACM SIGCOMM Computer Communication Review*, vol. 36, no. 4, pp. 231–242, Oct. 2006.

[8] M. Roughan, "Fundamental Bounds on the Accuracy of Network Performance Measurements," *ACM SIGMETRICS Performance Evaluation Review*, vol. 33, no. 1, pp. 253–264, Jun. 2005.

[9] T. R. Henderson, M. Lacage, G. F. Riley, G. Dowell, and J. B. Kopena, "Network Simulations with the ns-3 Simulator," in *Proceedings of ACM SIGCOMM 2008*, Seattle, WA, USA, Aug. 2008, p. 527.